ON THE DESIGN OF SAW-FILTERS WITH BUTTERWORTH PASSBAND AND CHEBYSHEV STOPBAND CHARACTERISTICS

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Abstract

Butterworth filter synthe is suffers from too gradual a transition from passband to stopband while Chebyshev approximation gives rise to passband ripple.

In this paper we will describe how to modify the well known Park-McClellan approximation algorithm which is widely used to synthesize Chebyshev transfer functions to yield a filter transfer function with Butterworth characteristics in the passband and Chebyshev (equiripple) behaviour in the stopband.

Experimental results will be presented and possible improvements will be discussed.

1. Introduction

In some applications even small passband amplitude or phase ripple can be troublesome. This paper will introduce a modification of the well known Parks-McClellan algorithm to yield a maximally flat filter transfer function.

In a first paragraph the basics of the Parks-McClellan algorithm are revised. Next, the necessary modifications pertinent to our design are introduced and the consequences are assessed. Finally, experimental data is compared to calculated results and design improvements (theoretical and practical) are discussed.

2. Finite Impulse Response Filter

A transversal filter has a finite impulse response h(k) the z transform of which is given by /1/

$$H(z) = \sum_{k=0}^{2N} h(k)\, z^{-k} \qquad (1)$$

H(z) is a polynomial of the order 2N with 2N zeros dispersed in the complex z plane. The additional 2N poles at the origin do not contribute to the transfer function because this transfer function is the Fourier transform of h(k) which is equal to the z transform evaluated along the unit circle:

$$H(e^{j\omega}) = \sum_{k=-N}^{N} h(k)\exp(-jk\omega)\exp(-jN\omega) \qquad (2)$$

Now, the condition for linear phase is

$$h(k) = h(-k) \qquad k=1,\dots,N \qquad (3)$$

Equation (3) inserted into (2) and omitting $\exp(-jN\omega)$ which is just a constant signal delay will yield

$$H(\omega) = h(0) + 2\cdot\sum_{k=1}^{N} h(k)\cos(k\omega) \qquad (4)$$

which is a real valued function.

Substituting

$$\cos(k\omega) = \sum_{m=0}^{k} d_m(\cos\omega)^m \qquad (5)$$

and

$$x = (1-\cos\omega)/2 \qquad 0 \le x \le 1 \qquad (6)$$

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will transform ④ into a N-th degree polynomial H(x):

$$H(x) = \sum_{k=0}^{N} a_k x^k \qquad 0 \leq x \leq 1 \qquad (7)$$

This polynomial will be used to approximate the filter transfer function using the Parks-McClellan /2/ algorithm.

3. The Modified Hofstetter Algorithm

The method introduced in this paper is based on the Hofstetter algorithm /1,4/ which is a version of the above mentioned Parks-McClellan equiripple design.

This procedure will yield a maximally flat passband response by equating the first (s-1) derivatives of the approximation polynomial H(x) ⑦ to zero at the origin x=0 (ω=0)):

$$\frac{\partial^m}{\partial x^m} H(x)\Big|_{x=0} = 0 \qquad m=1,\ldots,s-1 \qquad (8)$$

and subsequently approximating the stopband in the Chebyshev sense with a constant ripple δ (Fig. 1).

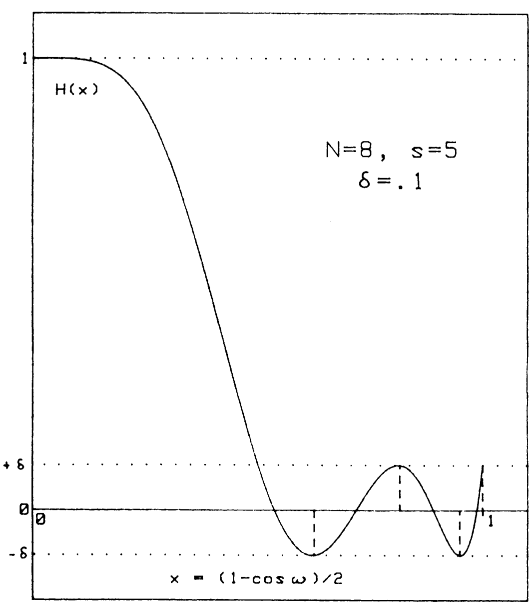

Fig. 1 Lowpass filter with maximally flat passband and equiripple stopband

It can be seen from Fig. 1 the value of the transfer function is equal to one at the origin

$$H(x)\Big|_{x=0} = 1 \qquad (9)$$

Given the filter degree N (degree of polynomial ⑦) with a given stopband error δ , there are exactly N different optimal filters corresponding to the number s-1 of vanishing derivatives, where

$$1 \leq s \leq N.$$

Such a family of filters is shown in Fig.2.

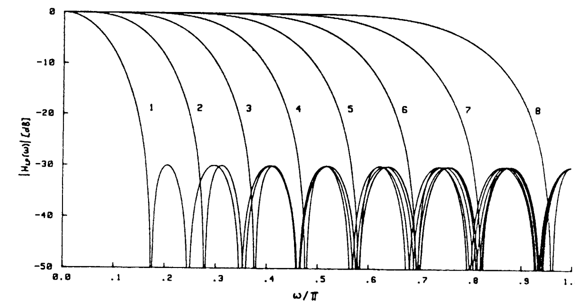

Fig. 2 Family of lowpass filters with flat passband and equiripple stopband (N=8, =-30 dB,s=1,...,8)

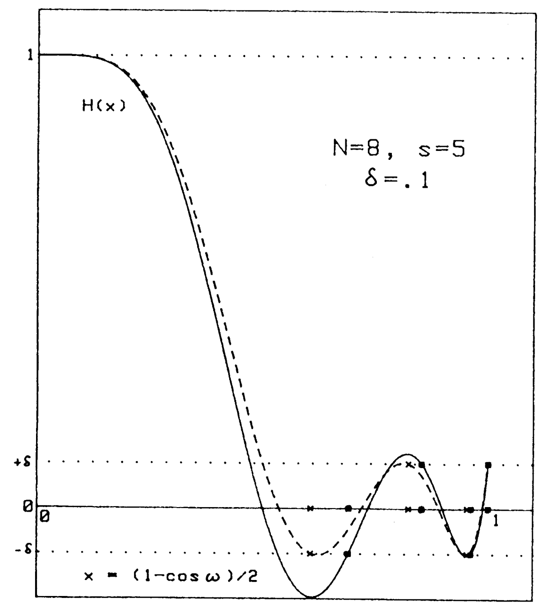

Fig. 3 Succesive approximation in our design procedure.

Applying ⑧ and ⑨ on the polynomial H(x) ⑦ yields

$$H(x) = 1 + x^s \sum_{k=0}^{N-s} a_{k+s} x^k \quad (10)$$

with N-s+1 unknown coefficients. This polynomial can have at most N-s local extrema in the interval $0 \leq x \leq 1$ and an additional extremum at the edge x=1.

The design will now proceed analogous to the Hofstetter algorithm /4,1/, consecutive steps of which are depicted in Fig. 3.

First, the positions x_i of the N-s+1 extrema taking on values

$$H(x_i) = (-1)^{i-s+1} \cdot \delta \quad ; i=s,..,N; \; x_N = 1$$

are estimated (represented by dark squares in Fig. 3). Next, a hermite interpolation polynomial is fitted to the data points x_i. Any intermediate point of H(x) can now be computed (solid curve in Fig.3).

This computation is most efficiently performed by using the barycentric form of the interpolation polynomial, which is insensitive to numerical rounding problems:

$$H(x) = \frac{\displaystyle\sum_{m=1}^{s} \frac{c_m}{x^m} + \sum_{i=s}^{N} \frac{b_i}{x-x_i} \cdot H(x_i)}{\displaystyle\sum_{m=1}^{s} \frac{c_m}{x^m} + \sum_{i=s}^{N} \frac{b_i}{x-x_i}} \quad (11)$$

with

$$b_i = x_i^{-s} \prod_{\substack{k=s \\ k \neq i}}^{N} \frac{1}{(x_i - x_k)}$$

with i = s,...,N

and the recursivly defined coefficients

$$c_s = \prod_{i=s}^{N} \frac{1}{(-x_i)}$$

and

$$c_m = \frac{1}{(s-m)} \cdot \sum_{k=m+1}^{s} c_k S_{k-m}$$

with m = 1,...,s-1

and the auxiliary sums

$$S_k = \sum_{i=s}^{N} \frac{1}{x_i^k}$$

with k = 1,..,s-1

The actual extrema can thus be found by evaluating the interpolation formula on a closely spaced grid of points x_j and subsequently be taken as the new estimates of the real extrema for the next iteration (represented by crosses in Fig. 3). The algorithm stops when all extrema have the prescribed value $\pm\delta$.

As in the equiripple design, the passband and stopband edges are free to move during subsequent iterations and will only be fixed after the final solution is found.

Finally, the weighting factors h(k) of the taps of equation 1 are obtained by sampling $H_{LP} = H((1-\cos\omega)/2)$ and performing the inverse Fourier transform.

4. Experiment

To test the modified procedure, the following SAW filter was designed:

center frequency	90 MHz
fractional bandwidth	16.2 MHz = 18 %
stopband rejection	70 dB

The filter should consist of two identical interdigital transducers with split electrodes coupled by a multistrip coupler (MSC). The number of electrodes has been chosen to be 51 to keep computation time rather short. Thus a filter degree of N = 25 was realized. Setting the first 3 derivatives of the approximation polynomial to zero (s=4) would yield a fractional bandwidth of 18 %. With 70 dB of stopband rejection, the transition width was found to be 5.2 % or 4.7 MHz.

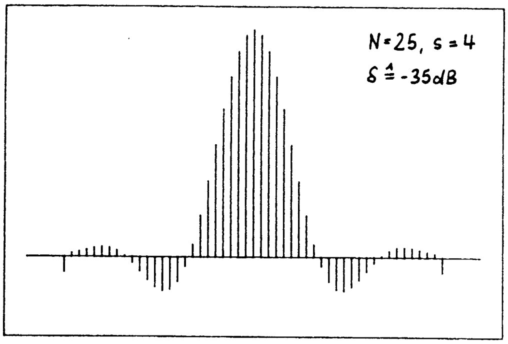

Fig. 4 Weighting factors h_k

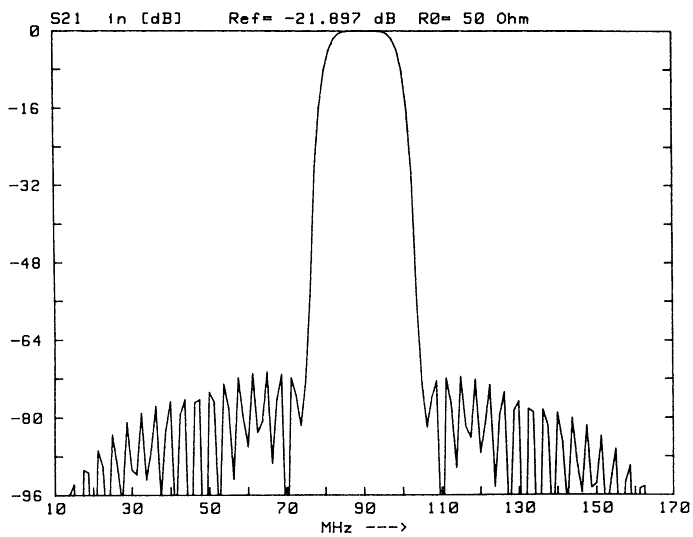

Fig. 5 Computed transfer function

Fig. 4 shows the resulting tap weighting factors h_k computed wih the above introduced algorithm which correspond to the overlapping length of the electrodes of our SAW filter.

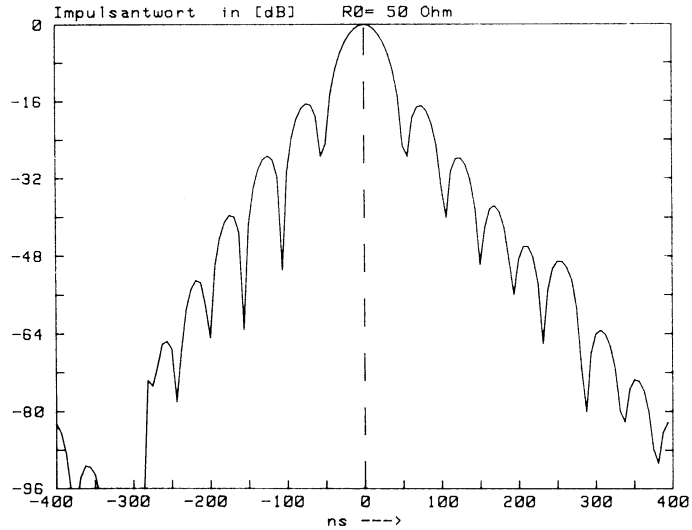

Fig. 6 Computed impulse response (FFT of transfer function of Fig. 5)

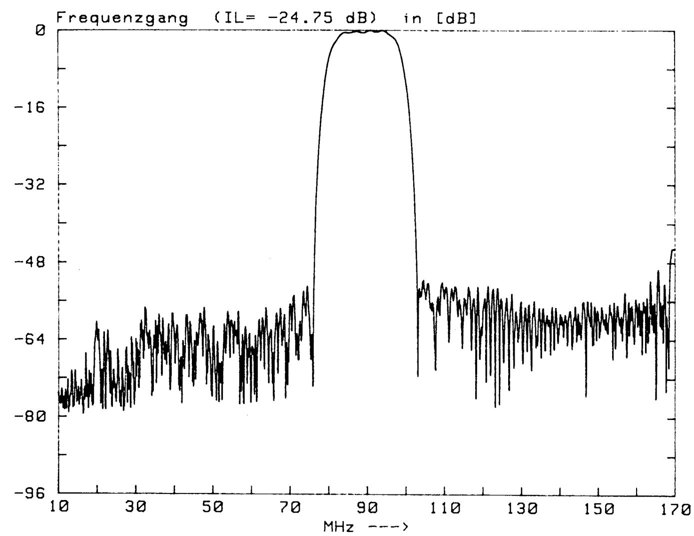

Fig. 7 Measured frequency response

Fig. 5 shows the computed frequency response of the unmatched SAW filter in a 50 Ω - system. The calculation was performed using the Mason cross field equivalent circuit /5/ neglecting the broadband response of the multistrip coupler, the triple transit signal and all effects caused by diffraction.

Fig. 6 shows the computed impulse

response calculated from the transfer function by means of a FFT (Fast Fourier Transform).

Comparison of the computed transfer function (Fig. 5) with the measurement (FIG. 7) immediately shows two discrepancies:

1 - the experiment shows passband passband ripple while theory pre dicts none
2 - the specified stopband rejection of 70 dB could not be obtained

5. Discussion

It is assumed that the ripple is caused by reflections of the SAW at the ends of the interdigital transducers. To proof this assumption, a composite impulse response including this end reflection has been modelled (Fig. 8).

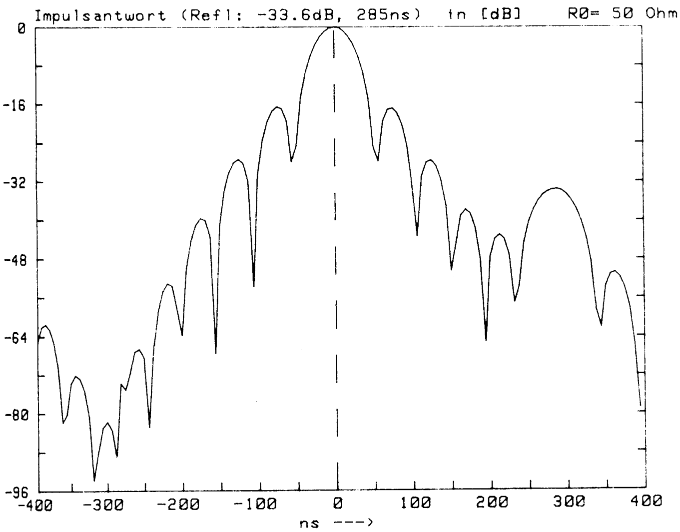

Fig.8 Composite impulse response including end reflections

A subsequent FFT shows excellent agreement with the experiment (Fig. 9).

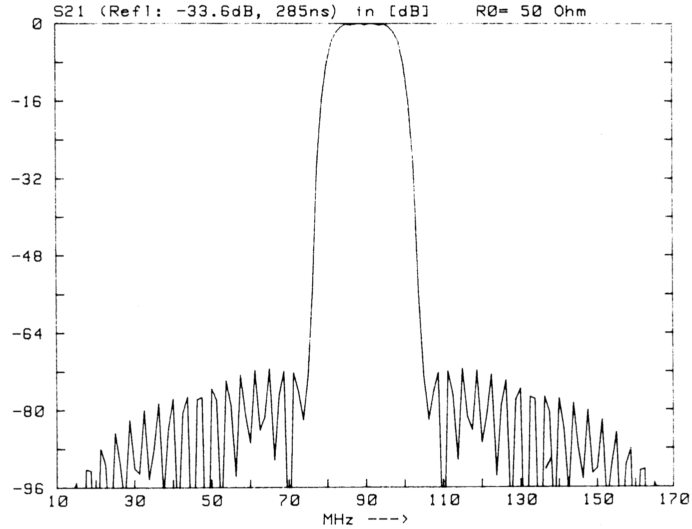

Fig. 9 Computed transfer function including end reflections

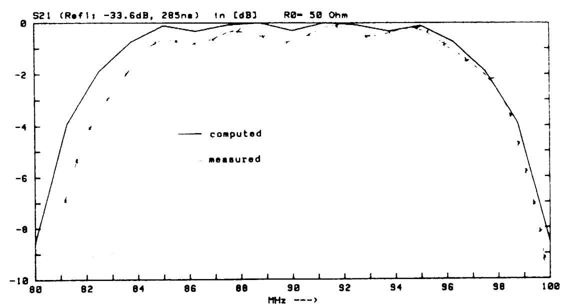

Fig. 10 Close-up of passband of Fig. 9

7. Conclusion

It could be shown that the modified Hofstetter algorithm introduced in this paper is a viable method fo synthesizing maximally flat SAW bandpass filters with constant stopband rejection.

8. Acknowledgments

Acknowledgments are due to the members of the Central Research Laboratory of SIEMENS AG, Munich, West Germany for producing the prototypes.

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